The UR Strategy in Str8ts puzzles

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by BP

Introduction

This tutorial is a guide how to use the UR Strategy for solving (extreme) Str8ts puzzles.

I hope you‘ll find this text helpful. If you have comments or corrections please mail to BP.Str8ts@web.de.

Many thanks to Burkhard and Ben for their helpful feedback!

Uniqueness assumption

When solving Str8ts puzzles, we usually make an assumption we don't think about any further: If two players solve the puzzle correctly and independently of each other – using the known strategies or other strictly logical steps – then both will arrive at the same solution. We assume that there is only one solution, i.e. that this solution is unique.

Strictly speaking, this is not permissible. The definition of Str8ts puzzles does not require unique solutions. Nevertheless, it is common for puzzles to have only one solution. Typically this applies to all puzzles found in newspapers or magazines, it certainly applies to all puzzles on www.str8ts.com.

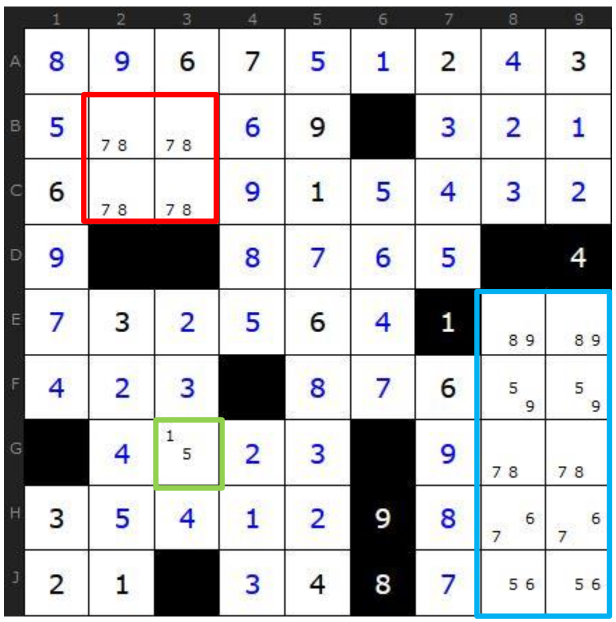
In his excellent paper “Str8ts Strategies” SlowThinker calls these puzzles “well-designed”. Having a well-designed puzzle, i.e. one with a unique solution, has several advantages, for example people are able to discuss and compare their paths to the solution. And there’s another advantage: Knowing that a Str8ts puzzle has a unique solution may help find this solution. There’s a solving strategy based on the uniqueness assumption. This strategy is called Uniqueness Argument, Unique Solution Constraint, or Unique Rectangle Rule, and is typically abbreviated to UR.

This tutorial explains the UR strategy and gives a list of examples how to use it.

Puzzles with multiple solutions

To understand how the UR strategy works in well-designed puzzles let‘s look at a not so well-designed puzzle.

Example 0:



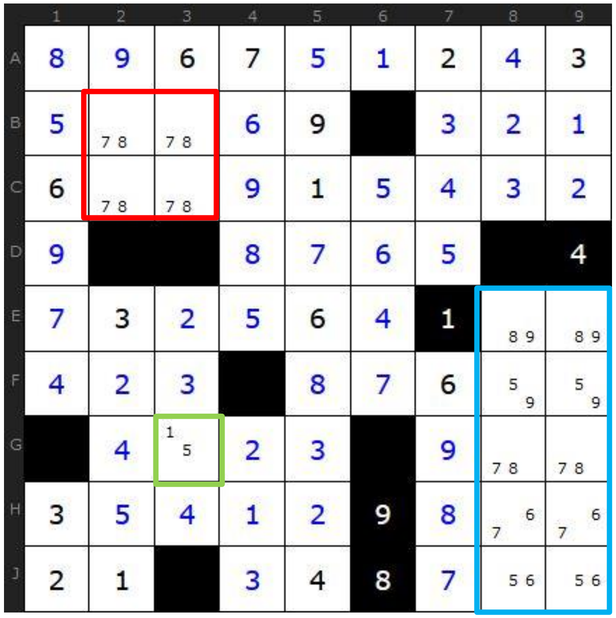
Example 0 looks like a “normal” almost completely solved str8ts puzzle. The black digits are the original clues, the blue digits were entered by the solver.
But if you look at the unsolved cells you will not be able to determine which digits to fill in.

Look at **BC23**: Obviously there are two possible solutions: B2=C3=7, B3=C2=8 or vice versa B2=C3=8, B3=C2=7. So which is the correct one? Both solutions are correct because both fulfill the **conditions required by the Str8ts rules:**

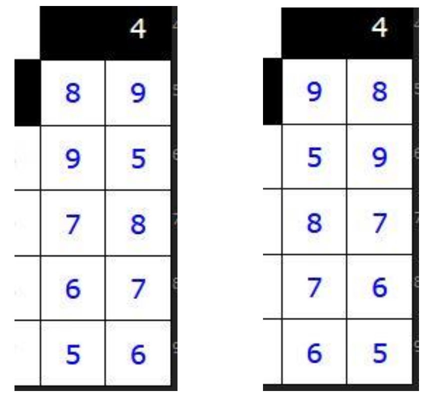
(1) Each digit occurs at most once in each row/column.
(2) The digits in the compartments form straights.
(3) The white on black clues don‘t occur in the corresponding rows/columns.

Puzzles with multiple solutions

Example 0 (continued):



In **EFGHJ89** there are also two possible solutions and we cannot decide which is the correct one because both solutions would be fulfilling the Str8ts conditions.



Note: We can derive the second from the first solution by swapping EFGHJ8 with EFGHJ9, i.e. swapping E8 with E9, F8 with F9, …, J8 with J9.

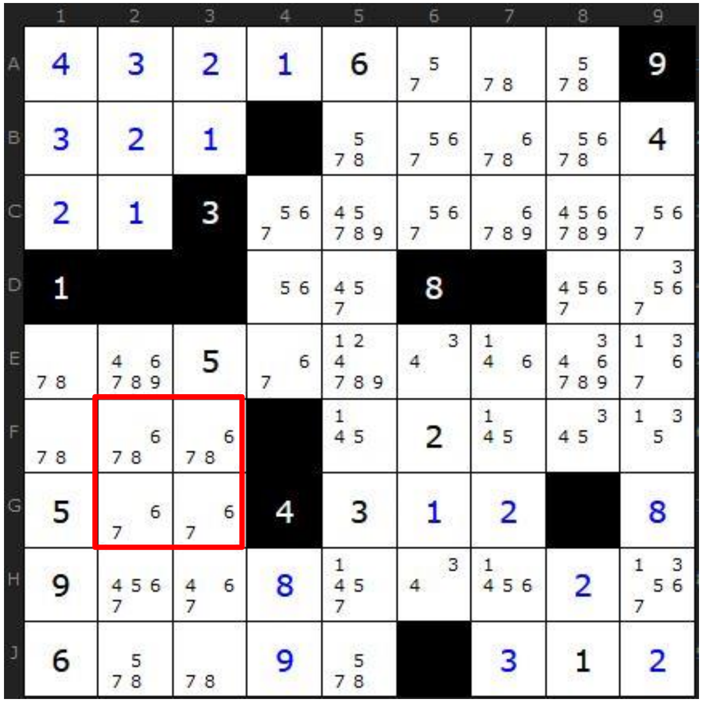
In **G3** there are two possible valid solutions as well: G3=1 and G3=5.

So this puzzle has not one, but **8 solutions**:
2 possibilities for BC23, EFGHJ89, G3 each => 2*2*2=8.

UR – First Example

In the above not well-designed example it is not possible to decide on a unique solution. The following example shows how that helps us in solving well-designed Str8ts puzzles. Note that we may assume the following puzzle is well-designed (i.e. has a unique solution), since it is the weekly extreme #711 on www.str8ts.com.

Example 1 (#711):



Look at **FG23:** What would happen if F1=8?
F1=8 => F2=G2=F3=G3=67 (in short: FG23=67)

This is exactly the same scenario that we encountered in BC23 of Example 0. This means: If we set F1=8 we will **not** find a unique solution to the puzzle, there will be at least two. One with F2=G3=6,F3=G2=7, one with F2=G3=7,F3=G2=6. Since we know the solution to be unique we may conclude that F1<>8 (which implies F1=7).

UR – Recap

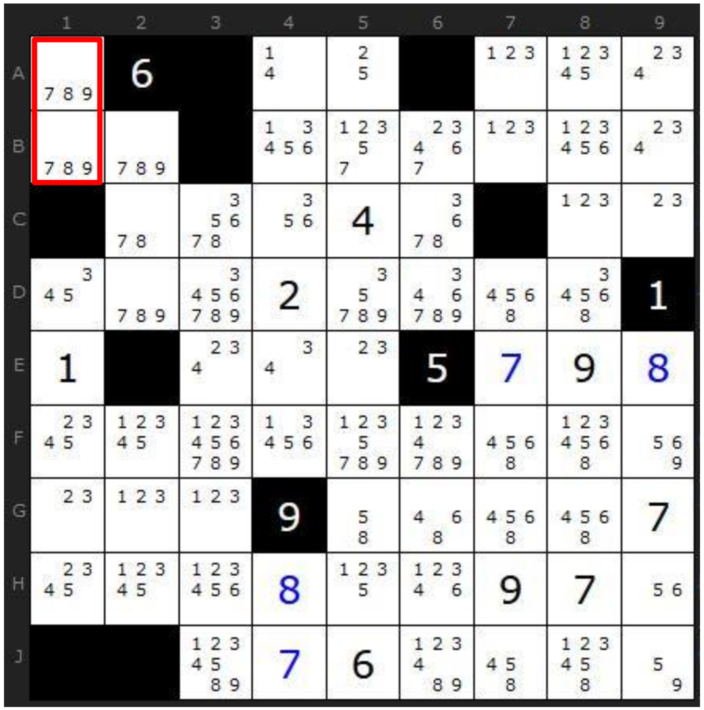
To recap: All the puzzles we usually encounter are well-designed so we may assume that they have exactly one solution. In these puzzles scenarios that lead to multiple solutions (for example those described in Example 0) are forbidden and must be avoided. Drawing conclusions on this basis is called using the UR Strategy or just using an UR.

The remainder of this tutorial is a list of examples to show some of the situations in which a UR is useful. These examples are supposed to help you get a feeling for how to apply URs. If you don‘t immediately understand one example don‘t give up. Leave it out and try the next. Many explanations will be repeated in several examples, but sometimes phrased differently to give you several chances to understand them.

Note: We assume that all puzzles in the remainder of this tutorial are well-designed.

Example 2

Example 2 (#757):

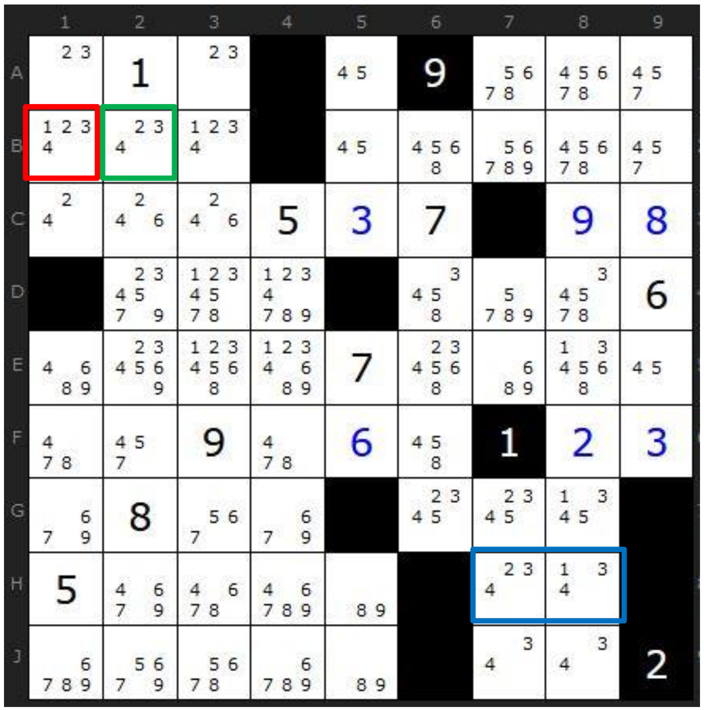


Look at **AB1**:
If B1=8 then A1=79 and we couldn't decide whether A1 is 7 or 9. Both options would be valid.
Therefore B1=79 (which implies A1=B2=8).

Note: This argument works because: If B1=8 then A1 is the only possible cell for 7 and 9 in row A as well as in col 1.

Example 3

Example 3 (#760):



Look at **B1**:
If B1=4 yielded a valid solution then replacing it by B1=1 would yield a second valid solution in contradiction to our assumption of uniqueness. Therefore B1<>4.

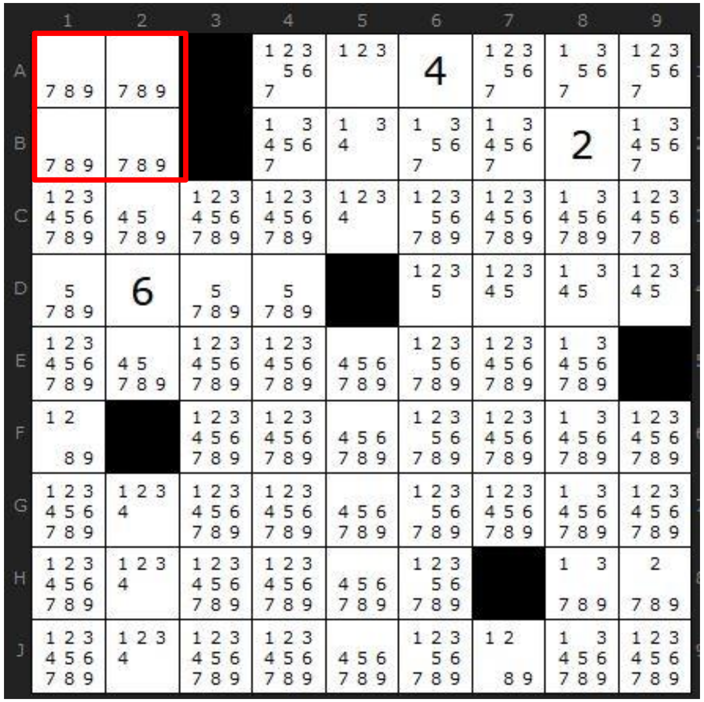
Note: This argument works because: If B1=4 then 1 is neither in row B nor in col 1.
Note: At this point we cannot conclude that B1<>1.

Look at **B2**:
If B2=4 then AB13=23 and we cannot decide whether A1=B3=2 and A2=B1=3 or vice versa. Thus we would get two solutions in contradiction to our assumption of uniqueness. Therefore B2<>4.

Also by UR: **H78**<>34 and therefore 4 can be removed from H78.

Example 4: URs may help with Setti‘s rule

Example 4*:

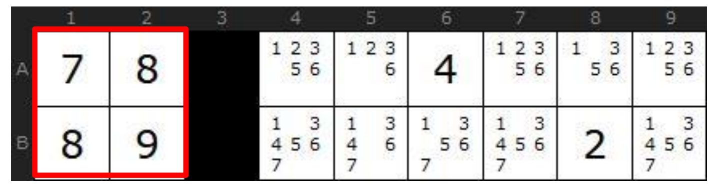


AB12: By UR obviously the following is true:

- A12 and B12 cannot both be 78.
- A12 and B12 cannot both be 89.

Therefore one pair is 78, the other is 89.

Now assume A12=78, B12=89:



Then 7 **must be** in B456789, otherwise we could swap A12 and B12 getting A12=89, B12=78, which would also be valid.

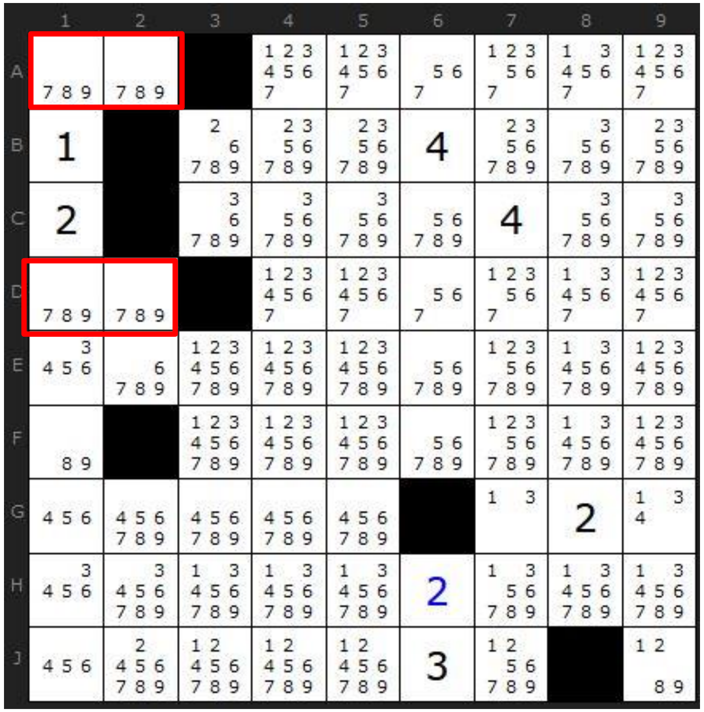
The same holds if A12=89 and B12=78. Then 7 must be in A456789.

So, in situations like this we can conclude:
One pair of A12, B12 is 78, the other is 89, and 7 belongs to A and B.
The latter may be valuable information for applying Setti‘s rule to 7.

(*) Note that this is not a valid puzzle, just a pattern used for demonstration purposes.

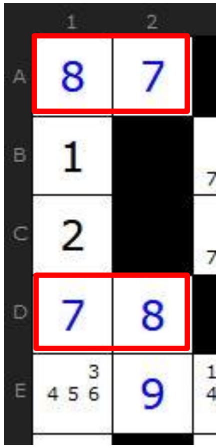
Example 5

Example 5*:



The situation here may at first glance look similar to the one in the last example, but there is a big difference: In this case we cannot conclude that of the pairs **A12** and **D12** one is 78, the other 89 because D2 depends on E2.

For example:
If A1=8,A2=7,D1=7,D2=8,E2=9 then we cannot just swap A12 with D12 because then DE2=79 wouldn‘t be a straight anymore.
So A12=D12=78 is possible here.

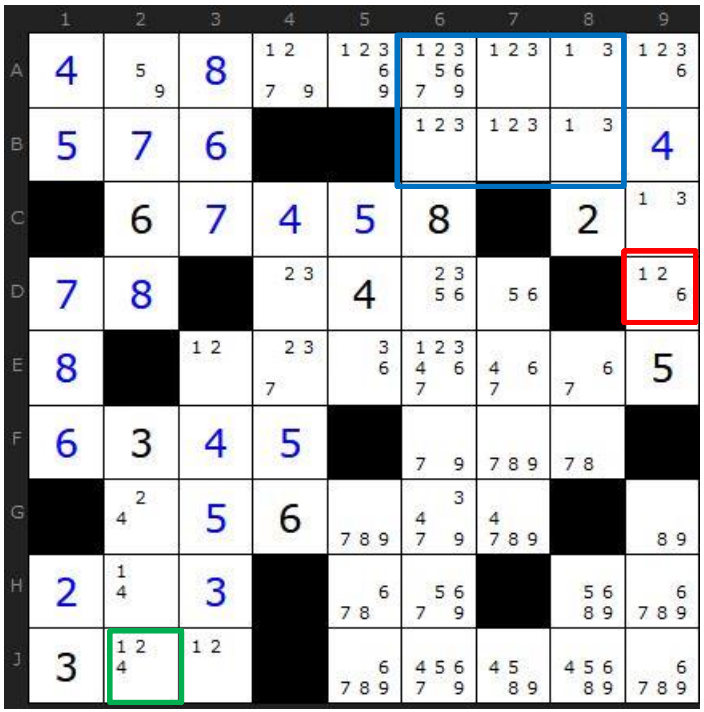


Conclusion: Be very careful when applying the “swapping argument”. If you argue that by changing/replacing/swapping digits within cells you get another valid solution make sure that this other solution is indeed feasible!

(*) Note that this is not a valid puzzle, just a pattern used for demonstration purposes.

Example 6

Example 6 (KST_682):



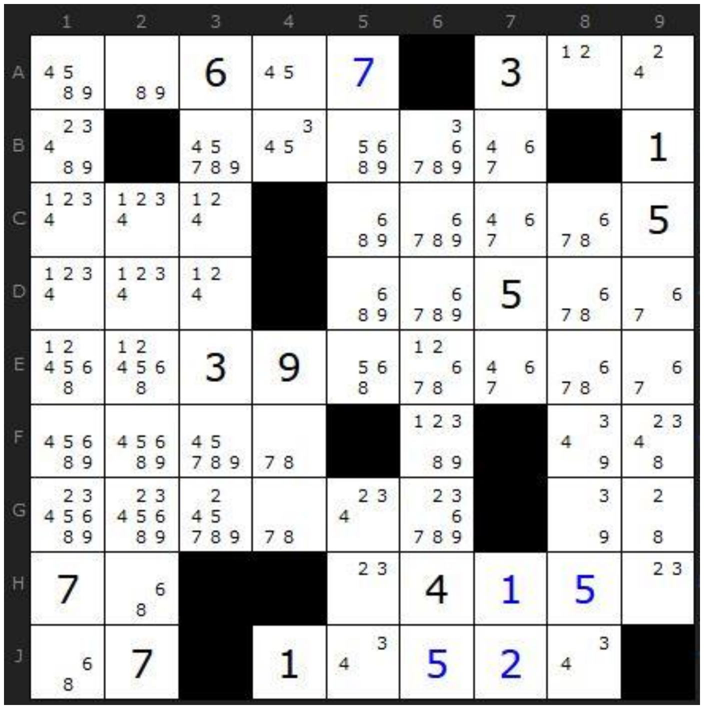
D9: D9<>6, otherwise we could replace it by D9=1.
Also: If D9=1 then 6 must be in D67 or HJ9 (or both) because otherwise we could replace D9=1 by D9=6.

AB678: A6<>123, otherwise in the solved puzzle we could swap A678 with B678 thus creating a second solution.
Note: This swap wouldn‘t hurt the Str8ts conditions because the digits in the involved compartments (A1-9, B6789, A-J6, AB7, ABC8) wouldn‘t change. Only their order would change.

J2: J2<>4, otherwise H2=1, J3=2 and we could swap J2 with H2.
Also: If J2=1 then 4 must be in J678 because otherwise we again could swap J2 with H2.

Example 7: URs and fishes

Example 7 (KST_684):



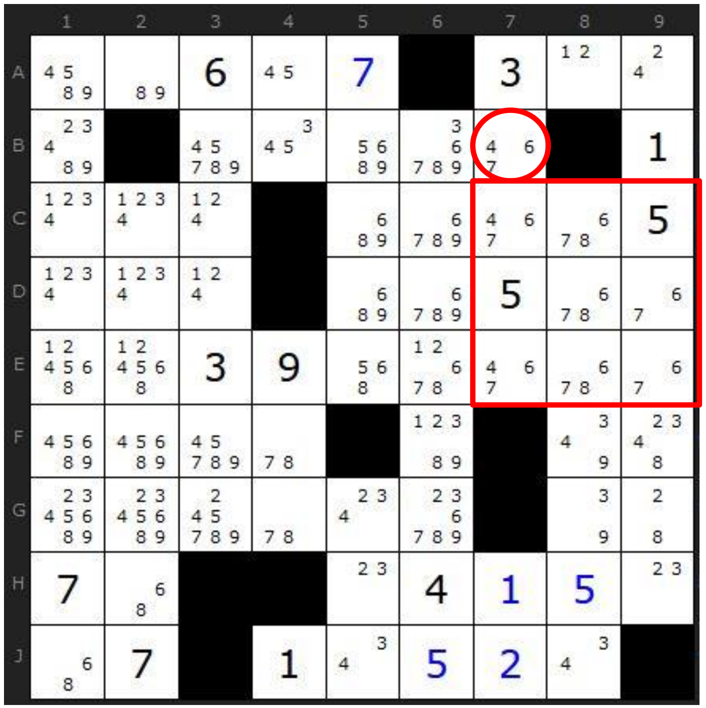
This example contains several URs:

- C8<>8, otherwise DE89=67.
- 2 must be in CD3, otherwise CD12=23.
- One of triples C123 and D123 is 123, the other is 234, and 4 must be in rows C and D. Since 4 is not possible in D56789 this leads to: C123=123, C7=4, D123=234.

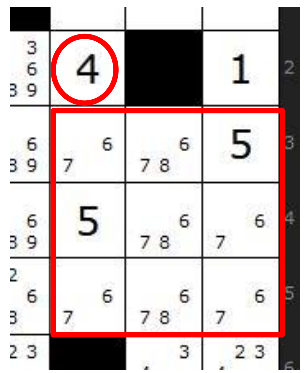
The above URs are a repetition of UR steps you‘ve already seen. On the next slide I show you a new UR step. To that end I assume we haven‘t applied the above UR steps, i.e. that we do not know yet that C7=4.

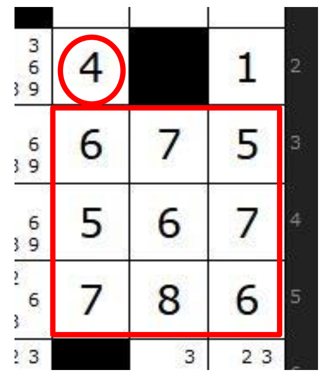
Example 7: URs and fishes

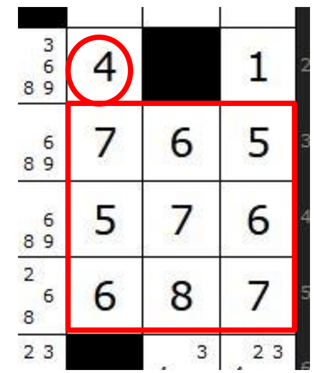
Example 7 (KST_684), continued:



A new UR step: Look at **B7 and CDE789**:
If B7=4 then we get a swordfish on 6 and one on 7 in 789CDE and for any given solution we get a second one by swapping all 6‘s and 7‘s within 789CDE.
Therefore: **B7<>4**.

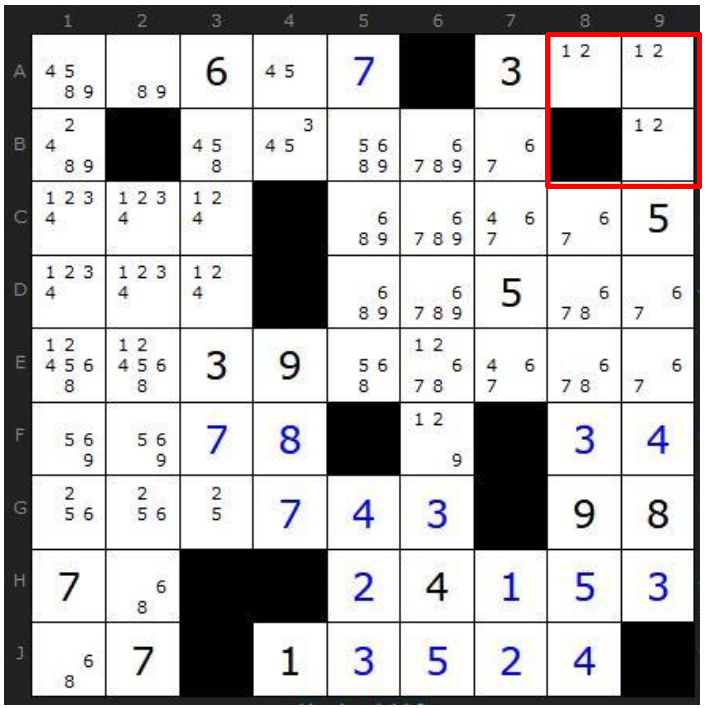




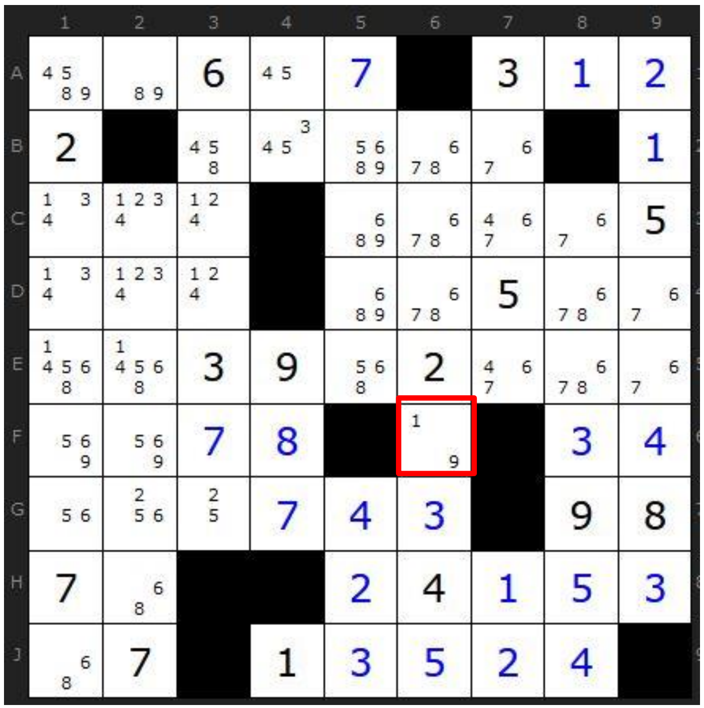


Example 8 + 9: It‘s your turn

Example 8 (KST_684 modified):



Example 9 (KST_684 modified):



Any idea how to apply the UR rule to the marked regions in these examples? Solution below.

Example 8: B1=2, otherwise two valid solutions for A89,B9: A8=B9=1,A9=2 and A8=B9=2,A9=1.

Example 9: 9 in F12, otherwise F6 not uniquely determined: 1 and 9 would both be feasible.

Example 10: It‘s your turn

Example 10*:

	1	2	3	4	5	6	7	8	9
A	9	8		2 3 4 5 6	2 3 4 5 6	2 3 4 5 6	2 3 4 5 6	7	3 4 5 6
B	5 6 7	1 2 3 4 5 6 7	1 2 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	2 3 4 5 6 7	8	9
C	8	2 3 4 5 6 7 9	2 4 5 6 7 9	2 3 4 5 6 7 9	2 3 4 5 6 7	2 3 4 5 6 7 9	2 3 4 5 6 7 9		1 2
D	5 6 7	3 4 5 6 7	4 5 6 7 8	3 4 5 6 7 8	9	3 4 5 6 7 8	3 4 5 6 7 8		1 2
E	5 6 7	1 2 3 4 5 6 7 9	1 2 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8 9	2 3 4 5 6 7 8 9	3 4 5	3 4 5 6 7 8
F		1 2 3 4 5 6 7 9	1 2 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8 9	2 3 4 5 6 7 8 9	3 4 5	3 4 5 6 7 8
G	1 3 4	1 3 4 5 6 7 9	1 4 5 6 7 8 9	1 3 4 5 6 7 8 9	1 3 4 5 6 7 8	1 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2	3 4 5 6 7 8
H	1 2 4	1 2 4 5	3	1 2 4 5	1 2 4 5		7 8	6	7 8
J	2 3 4	2 3 4 5 6 7 9	2 4 5 6 7 8 9	2 3 4 5 6 7 8 9	2 3 4 5 6 7 8	2 8 9	1	3 4 5	3 4 5 6 7 8

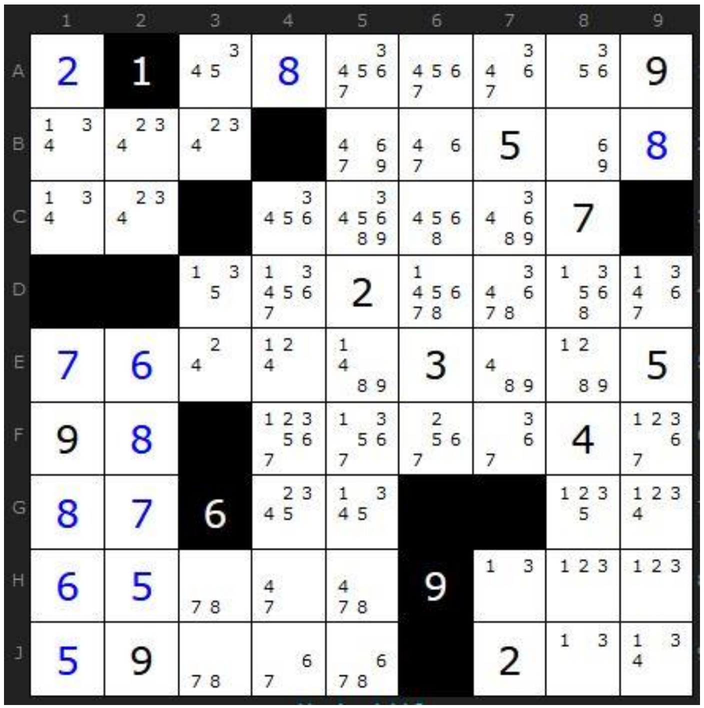
Any idea how to apply the UR rule to the marked regions in this example? Solution below.

(*) Note that this is not a valid puzzle, just a pattern used for demonstration purposes.

F3<>1,9, otherwise you replace 1 by 9 or vice versa.
CD9: If 2 not in C2-7 then we get two valid solutions C9=1,D9=2 and C9=2,D9=1. Therefore 2 in C2-7 which implies C9=1,D9=2.

Example 11: Clearing a whole area with consecutive URs

Example 11 (#702):



7 not in HJ5, otherwise HJ35=78.

B1<>4, otherwise it may be replaced by 1.

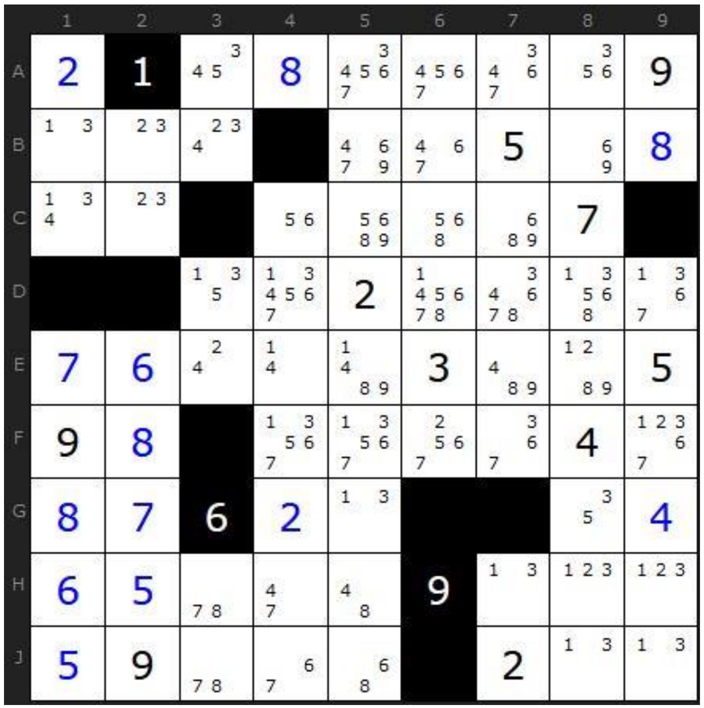
C2<>4, otherwise it may be replaced by 2.
B2<>4, otherwise BC12=34.
Note: These two URs remove 4 from col 2.

4 in B because:
If B1=1 then 4 in B568, otherwise we could replace B1=1 by B1=4.
If B1<>1 then 4 in B123.
Therefore 4 always in B.

D9 <> 4 because: D9=4 would remove 4 as a candidate from GJ9 and 5 from G8. Thus GHJ8 and GHJ9 would only contain 1,2,3 and we could swap GHJ8 with GHJ9.

Example 11: Clearing a whole area with consecutive URs

After the URs described above and several other steps Example 11 looks like this:
Example 11 (#702), continued:



C2 <> 2, otherwise we could replace it by 4.

Note: I'd like to indicate a point some players might find irritating: On the last slide we used two URs to remove 4 from col 2 altogether. So: If 4 is already deleted from col 2 how can we now argue that we could replace C2=2 by C2=4?

We can because the argument in length works as follows: If C2=2 (and hence C1=B1=3) was part of a valid solution then we could construct another valid solution by replacing C2=2 by C2=4. Since two valid solutions contradict our uniqueness assumption C2=2 must be false.

Conclusion

I hope this tutorial helped you to (better) understand the UR Strategy and gave you some ideas how to use it – if you are willing to do so. Some purists reject the UR Strategy because it relies on the additional assumption that a puzzle is unique.

URs come in all shapes and sizes, sometimes they are simple one-cell or standard rectangular URs, sometimes they need a clever extra idea to work. This tutorial did not by far cover all situations in which an UR might be useful. It's purpose was to give you a starting point.

Thanks for your attention!
Petra (aka P in BP)